Name: Solutions

Date: 4/1/2025

Math 130
Quiz 10

Formulas you may need: $\mu_{\bar{X}} = \mu_X$ $\sigma_{\bar{X}} = \dfrac{\sigma_X}{\sqrt{n}}$ $\mu_{\hat{p}} = p$ $\sigma_{\hat{p}} = \sqrt{\dfrac{pq}{n}}$

1. (2 points) State the central limit theorem (for $\bar{X}$).

If X is any random variable and $n \geq 30$, then $\bar{X}$ has a normal distribution

If X has a normal distribution to begin with, then $\bar{X}$ has a normal distribution no matter what n is.

$$\mu_{\bar{x}} = \mu_x \quad , \quad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

2. (4 points) 8.3% of all people have asthma. In a group of 300 people, what is the probability that between 10% and 12% of them will have asthma?

X = The yes or no answer when a randomly selected person is asked if they have asthma.

Dist: unknown

$p = 0.083$
$q = 0.917$

$\hat{p}$ = The percentage in a randomly selected group of 300 people who have asthma.

Condition: $npq \geq 10$?

$(300)(0.083)(0.917) = 22.8333 \geq 10$ ✓

Dist: Normal (by CLT bec. $npq \geq 10$)

$\mu_{\hat{p}} = p = 0.083$

$\sigma_{\hat{p}} = \sqrt{\dfrac{pq}{n}} = \sqrt{\dfrac{(0.083)(0.917)}{300}}$

$$P(0.10 < \hat{p} < 0.12) \overset{\text{Z-trans}}{=} P\left(\frac{0.10 - \mu_{\hat{p}}}{\sigma_{\hat{p}}} < \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} < \frac{0.12 - \mu_{\hat{p}}}{\sigma_{\hat{p}}}\right)$$

$$= P\left(\frac{0.10 - 0.083}{\sqrt{\frac{(0.083)(0.917)}{300}}} < Z < \frac{0.12 - 0.083}{\sqrt{\frac{(0.083)(0.917)}{300}}}\right) = P(1.07 < Z < 2.32)$$

$$= P(Z < 2.32) - P(Z < 1.07) = 0.9898 - 0.8577 = \boxed{0.1321}$$

3. (4 points) The average time all people in the world spend on the internet per day is 402 min with a standard deviation of 53 min. If 145 people are randomly selected, what is the probability that they spend an average time of more than 395 minutes on the internet in a day?

X = The time a randomly selected person in the world spends on the internet in a day

Dist: Unknown

$\mu_X = 402$

$\sigma_X = 53$

$\bar{X}$ = The average time a randomly selected group of 145 people in the world spend on the internet in a day.

Dist: Normal (by CLT bec $n = 145 \geq 30$ ✓)

$\mu_{\bar{X}} = \mu_X = 402$

$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{53}{\sqrt{145}}$

$$P(\bar{X} > 395) \overset{\text{z-trans}}{=} P\left(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} > \frac{395 - \mu_{\bar{X}}}{\sigma_{\bar{X}}}\right)$$

$$= P\left(Z > \frac{395 - 402}{\frac{53}{\sqrt{145}}}\right) = P(Z > -1.59)$$

$$= 1 - P(Z < -1.59) = 1 - 0.0559$$

$$= \boxed{0.9441}$$